MATH1520AB 2021-22 Quiz 7 (week 11)

Full marks: 10 marks

Time allowed: 15 minutes

1. Evaluate $\int \frac{2x^3}{(x^2+1)^3}dx$ using the substitution $u = x^2 + 1$.

Answer.

Let $u = x^2 + 1$. Then $du = 2xdx$.

$$\begin{aligned}\int \frac{2x^3}{(x^2+1)^3}dx &= \int \frac{x^2}{(x^2+1)^3}2xdx \\ &= \int \frac{u-1}{u^3}du \\ &= \int (u^{-2} - u^{-3})du \\ &= -u^{-1} + \frac{u^{-2}}{2} + C \\ &= -\frac{1}{x^2+1} + \frac{1}{2(x^2+1)^2} + C \\ &= -\frac{2x^2+1}{2(x^2+1)^2} + C\end{aligned}$$

2. (a) Let $\frac{1}{x^2+4x-5} = \frac{A}{x+5} + \frac{B}{x-1}$. Find A and B.

   (b) Evaluate $\int \frac{1}{x^2+4x-5}dx$.

Answer.

(a) $1 = A(x-1) + B(x+5)$

Put $x = -5$. Then $1 = A(-5-1) \implies A = -\frac{1}{6}$.

Put $x = 1$. Then $1 = B(1+5) \implies B = \frac{1}{6}$.

(b) $\int \frac{1}{x^2+4x-5}dx = \int -\frac{1}{6(x+5)} + \frac{1}{6(x-1)}dx = -\frac{1}{6}\ln|x+5| + \frac{1}{6}\ln|x-1| + C$

3. (a) Let $\frac{2x-5}{4x^2-12x+9} = \frac{C}{2x-3} + \frac{D}{(2x-3)^2}$. Find C and D.

   (b) Evaluate $\int \frac{2x-5}{4x^2-12x+9}dx$.

Answer.

(a) $2x - 5 = C(2x-3) + D = 2Cx - 3C + D \implies C = 1, D = -2$

(b) $\int \frac{2x-5}{4x^2-12x+9}dx = \int \frac{1}{2x-3} - \frac{2}{(2x-3)^2}dx = \frac{1}{2}\ln|2x-3| + \frac{1}{2x-3} + C$